Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Discrete Mathematics

Subject Code : 4TE04DSM1 **Branch :** B.Tech. (CE,IT)

Semester : 4 **Date :** 24/04/2018 **Time :** 10:30 To 1:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Define a complete digraph.
b) Give an example of a weakly connected digraph having 4 edges.
c) Give an example of an acyclic digraph.
d) Define a group G.
e) Give an example of a non-commutative group.
f) Give an example of a cyclic group.
g) Define partially ordered set.
h) Write the least and greatest elements of the poset ({1, 2, 3, 5, 6, 10, 15, 30}, D).
i) Define a complete lattice.
j) Give an example of a sub algebra of the Boolean algebra B of divisors of 30.
k) Prove that if $a = b$, then $ab' + a'b = 0$.
l) Give an example of an infinite, proper subset of the set Q of rational numbers.
m) State the pigeon-hole principle.
n) Define a complement of a Fuzzy subset.

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) Let (L, ≤) be a lattice with binary operations $*$ and $\oplus$. Then, for any $a, b \in L$, prove that a ≤ b if and only if a $*$ b = a if and only if a $\oplus$ b = b . Explain the theorem by giving an example. **(07)**

b) Prove that $\langle S_{30}, D \rangle$ is a complemented lattice and also draw the Hasse diagram of it. **(07)**

Q-3 Attempt all questions **(14)**

a) For a lattice $\langle P(\{a,b,c\}),\subseteq\rangle$, answer the following questions: **(07)**

i) Find cover of each element (except the element {a, b, c}).

ii) Find lower bounds, upper bounds, greatest lower bound, least upper bound of $A=\{\{a\},\{a,b\}\}$.

iii) Find the least and greatest elements of it.

iv) Find atoms and anti-atoms of it.

b) Let $\langle L,*,\oplus,',0,1\rangle$ be a complemented lattice. Then, for any $a,b\in L$, prove that $a\le b \Leftrightarrow a*b'=0 \Leftrightarrow b'\le a' \Leftrightarrow a'\oplus b=1$. **(07)**

Q-4 Attempt all questions **(14)**

a) Prove that $(Z_6, +_6)$ is a group. **(05)**

b) Let G = Q^+, $a*b=\dfrac{ab}{2}$, then find the identity element and a^{-1} in G. **(04)**

c) Show that if a and b commute, then a^{-1} and b commute. Also, give an example of an abelian subgroup H in a non-abelian group G. **(03)**

d) Decide whether Z^*_{12} is a group or not under $\times_{12}$, the multiplication modulo 12. Give reason(s), if any. **(02)**

Q-5 Attempt all questions **(14)**

a) State and prove Stone's representation theorem. **(05)**

b) Obtain the sum of product canonical form of the Boolean expression x ⊕ y in three variables x, y, z. **(04)**

c) Define a Boolean algebra. Give an example of a lattice which is not a Boolean algebra. **(03)**

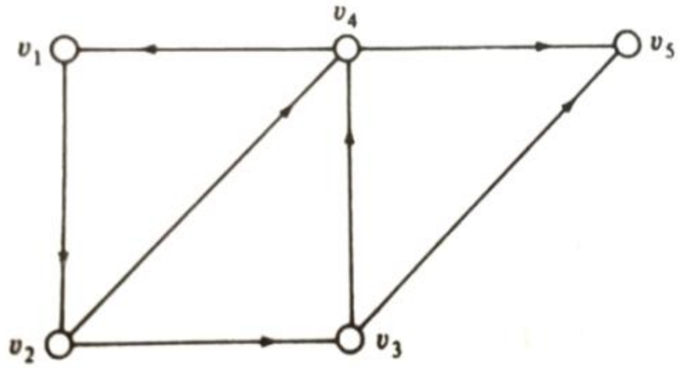
d) Prove that if $ab'+a'b=0$, then a = b. **(02)**

Q-6 Attempt all questions **(14)**

a) Show that the sum of indegrees of all the nodes of a simple digraph is equal to the sum of outdegrees of all its nodes, and that this sum is equal to the number of edges of the graph. Explain this statement by a simple example. **(05)**

b) From the graph given below, answer the following: **(05)**

1. Find in degree, out degree and total degree of each vertex.
2. Find reachable set of v_1.
3. Write any one node base.
4. Write the adjacency matrix from the below digraph. Consider the order v_1, v_3, v_2, v_4, v_5.



c) List the ways in which a directed tree can be represented graphically. Define a complete binary tree and give it's example. What is prefix code of the tree? **(04)**

Q-7 Attempt all questions **(14)**

a) Define: (1) predicate; (2) statement function; (3) quantifiers; and (4) free and bound variables. **(05)**

b) Give examples of: (1) predicate; (2) statement function; (3) quantifiers; and (4) free and bound variables. **(05)**

c) Symbolize the statement "given any positive integer, there is a greater positive integer" with and without universe of discourse. **(04)**

Q-8 Attempt all questions **(14)**

a) Let $E=\{a,b,c,d,e\}$, $\underset{\sim}{A}=\{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\}$, $\underset{\sim}{B}=\{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$ then find the following: **(07)**

1) $\underset{\sim}{A}\cup\underset{\sim}{B}$ 2) $\underset{\sim}{A}\cdot\underset{\sim}{B}$ 3) $\underset{\sim}{A}\hat{+}\underset{\sim}{B}$ 4) $\underset{\sim}{A}-\underset{\sim}{B}$ 5) $\underset{\sim}{A}\cap\underset{\sim}{B}$ 6) $\left(\underset{\sim}{A}'\right)'$ 7) $\underset{\sim}{B}'$

b) i) By using mathematical induction prove that $1+3+5+\ldots+(2n-1)=n^2$. **(07)**

ii) Solve the recurrence relation $a_{n+1} - 2a_n = 0$; $n \geq 0$ and $a_0 = 3$.